

Quark structure of $f_0(980)$ from the radiative decays $\phi(1020) \rightarrow \gamma f_0(980)$, $\gamma\eta$, $\gamma\eta'$, $\gamma\pi^0$ and $f_0(980) \rightarrow \gamma\gamma$

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 (20.03.2001)

Partial widths of the radiative decays $\phi(1020) \rightarrow \gamma f_0(980)$, $\gamma\eta$, $\gamma\eta'$, $\gamma\pi^0$ and $f_0(980) \rightarrow \gamma\gamma$ are calculated assuming all mesons under consideration to be $q\bar{q}$ states: $\phi(1020)$ is dominantly an $s\bar{s}$ state ($n\bar{n}$ component $\lesssim 1\%$), η , η' and π^0 are standard $q\bar{q}$ states, $\eta = n\bar{n}\cos\theta - s\bar{s}\sin\theta$ and $\eta' = n\bar{n}\sin\theta + s\bar{s}\cos\theta$ with $\theta \simeq 37^\circ$, and $f_0(980)$ is the $q\bar{q}$ meson with the flavour wave function $n\bar{n}\cos\varphi + s\bar{s}\sin\varphi$. Calculated partial widths for the decays $\phi(1020) \rightarrow \gamma\eta$, $\gamma\eta'$, $\gamma\pi^0$ are in a reasonable agreement with experiment. The measured value of the branching ratio $BR(\phi \rightarrow \gamma f_0(980))$ requires $25^\circ \leq |\varphi| \leq 90^\circ$; for the decay $f_0(980) \rightarrow \gamma\gamma$ the agreement with data is reached at either $77^\circ \leq \varphi \leq 93^\circ$ or $(-54^\circ) \leq \varphi \leq (-38^\circ)$. Simultaneous analysis of the decays $\phi(1020) \rightarrow \gamma f_0(980)$ and $f_0(980) \rightarrow \gamma\gamma$ provides arguments in favour of the solution with negative mixing angle $\varphi = -48^\circ \pm 6^\circ$.

13.40.Hq, 12.38.-t, 14.40.-n

I. INTRODUCTION

The problem of the $f_0(980)$ structure is actively discussed during the last decade, see for example [1–6] and references therein. The question is whether $f_0(980)$ belongs to the scalar quark nonet $1^3P_0q\bar{q}$ or whether it should be considered as an exotic state. Recent measurements of the radiative decay $\phi(1020) \rightarrow \gamma f_0(980)$ [7,8] have reinforced the discussion.

As was stressed in [4,9,10], it is reasonable to perform the $q\bar{q}$ nonet classification in terms of the K -matrix, or "bare", poles corresponding to states "before" the mixing which is caused by the transitions $q\bar{q} \text{ state} \rightarrow \text{real mesons}$. Such a mixing is crucial for the formation of the scalar/isoscalar resonances in the region 1200–1600 MeV, which are $f_0(1300)$, $f_0(1500)$ and the broad state $f_0(1530^{+90}_{-250})$. According to the K -matrix analysis, these resonances are formed as a result of mixing of the lightest scalar gluonium with the $1^3P_0q\bar{q}$ and $2^3P_0q\bar{q}$ states.

In terms of f_0^{bare} , the scalar/isoscalar states of the basic $1^3P_0q\bar{q}$ nonet are $f_0^{\text{bare}}(720 \pm 100)$ and $f_0^{\text{bare}}(1260 \pm 30)$; the state $f_0^{\text{bare}}(720)$ is close to the flavour octet $\varphi = -69^\circ \pm 12^\circ$ (the mixing angle is determined as $f_0 = n\bar{n}\cos\varphi + s\bar{s}\sin\varphi$ where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$), while the state $f_0^{\text{bare}}(1260)$ is almost a singlet ($\varphi = 21^\circ \pm 12^\circ$). The transitions which are due to the decay processes $f_0 \rightarrow \pi\pi$, $K\bar{K}$, $\eta\eta$ mix the states $f_0^{\text{bare}}(720)$ and $f_0^{\text{bare}}(1260)$ with each other as well as with nearby states: $f_0^{\text{bare}}(1235 \pm 50)$, $f_0^{\text{bare}}(1600 \pm 50)$, $f_0^{\text{bare}}(1810 \pm 30)$ (the orthogonality of the coordinate wave functions does not work in the transition $(q\bar{q} - \text{state})_1 \rightarrow \text{real mesons} \rightarrow (q\bar{q} - \text{state})_2$). Two of these states are the $2^3P_0q\bar{q}$ nonet members and one is the scalar gluonium, either $f_0^{\text{bare}}(1235)$ or $f_0^{\text{bare}}(1600)$: the K -matrix analysis cannot definitely tell us which one. Lattice calculations [11] favour $f_0^{\text{bare}}(1600)$ as a candidate for the glueball. After the mixing, we have in the wave $(IJ^{PC} = 00^{++})$ four comparatively narrow resonances ($f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_0(1750)$) and one rather broad state $f_0(1530^{+90}_{-250})$. The broad state ($\Gamma/2 \simeq 400 - 500$ MeV) appears as a result of a specific phenomenon which is appropriate for scalar/isoscalar states in the region 1200–1600 MeV, that is the accumulation of widths of overlapping resonances by one of them [12]. The analogous phenomenon in nuclear physics was discussed in [13]. Fitting to data proves that the main participants of the mixing are the glueball and two $q\bar{q}$ states belonging to the nonets $1^3P_0q\bar{q}$ and $2^3P_0q\bar{q}$, whose flavour wave functions are close to singlets: the transition $\text{glueball} \rightarrow q\bar{q}(\text{singlet})$ is allowed, while the transition $\text{glueball} \rightarrow q\bar{q}(\text{octet})$ is suppressed. Gluonium/ $q\bar{q}$ mixing is not suppressed according to the $1/N$ expansion rules [14] (see [4,15] for details). The glueball descendant is a broad state $f_0(1530^{+90}_{-250})$ which contains about 40–50% of the gluonium component; the rest is shared mainly by $f_0(1300)$ and $f_0(1500)$.

Following the K -matrix scenario, the resonance $f_0(980)$ is the descendant of $f_0^{\text{bare}}(720 \pm 100)$: the shift of the bare pole to the region of 1000 MeV is due to transitions of $f_0^{\text{bare}}(720 \pm 100)$ into real two-meson states, $\pi\pi$ and $K\bar{K}$. As to its origin, the state $f_0(980)$ is the superposition of states $q\bar{q}$, $q\bar{q}q\bar{q}$, $K\bar{K}$ and $\pi\pi$, and the wave function is to be represented by the correspondent Fock column. Concerning the $K\bar{K}$ and $\pi\pi$ components, one should take into account only the part of wave functions which respond to large meson-meson separations, $r_{\text{meson}} > R_{\text{confinement}}$; on the contrary, for the $q\bar{q}q\bar{q}$ state small interquark separations are to be taken into account. As to the glueball admixture in $f_0(980)$, the relative suppression of the decay $J/\Psi \rightarrow \gamma f_0(980)$ [16] tells us that it is not large.

To enlighten the problem of the content of $f_0(980)$, it would be reasonable to suggest a dominance of certain component: basing on the results of the K -matrix analysis [4,9,10] it is natural to check whether the hypothesis about

the dominance of $q\bar{q}$ component can explain the data on radiative decays $\phi(1020) \rightarrow \gamma f_0(980)$ and $f_0(980) \rightarrow \gamma\gamma$. This study is dictated by the existing opinion that measured widths of radiative decays contradict the interpretation of $f_0(980)$ as $q\bar{q}$ state, see mini-review [17] and references therein.

In Section 2 we calculate the decay amplitude $\phi(1020) \rightarrow \gamma f_0(980)$ assuming $f_0(980)$ to be a $q\bar{q}$ system. The technique for the description of composite $q\bar{q}$ systems and corresponding form factor calculations has been presented in detail in Ref. [18], where the pion form factor was studied as well as transition form factors $\pi^0 \rightarrow \gamma\gamma^*(Q^2)$, $\eta \rightarrow \gamma\gamma^*(Q^2)$ and $\eta' \rightarrow \gamma\gamma^*(Q^2)$. The method of spectral integration has been used for the form factor calculations. Within this technique, it is possible to introduce normalized wave functions. The transition amplitudes obey the requirement of analyticity, causality and gauge invariance. The used technique allows us also to perform calculations in terms of the light-cone variables.

It is worth noting that this calculation technique for the processes involving bound states has a broader applicability than for $q\bar{q}$ systems only: in [19] this method got its approbation by describing the deuteron as a composite np system, then this very technique was applied to heavy mesons [20,21]. For the convenience of a reader, in Section 2 we recall briefly the basic points of this approach and give necessary formulae for the calculation of the reaction $\phi(1020) \rightarrow \gamma f_0(980)$. Our calculations show that the data on branching ratio $BR(\phi \rightarrow \gamma f_0(980)) = (3.4 \pm 0.4_{-0.5}^{+1.5}) \cdot 10^{-4}$ [7,8,16] may be described within the $q\bar{q}$ structure of $f_0(980)$, with the mixing of $s\bar{s}$ and $n\bar{n}$ components as follows:

$$\psi_{flavour}(f_0(980)) = n\bar{n} \cos \varphi + s\bar{s} \sin \varphi, \quad 25^\circ \leq |\varphi| \leq 90^\circ. \quad (1)$$

The width $\Gamma(\phi \rightarrow \gamma f_0(980))$ depends strongly on the radius squared of $f_0(980)$, $R_{f_0(980)}^2$. At large $R_{f_0(980)}^2 \geq 12 \text{ GeV}^{-2}$ (0.47 fm^2), the data require $|\varphi| \sim 25^\circ - 45^\circ$, while at $R_{f_0(980)}^2 \sim 8 \text{ GeV}^{-2}$ (0.32 fm^2) one needs $|\varphi| \sim 40^\circ - 75^\circ$.

The estimation of radii of the scalar/isoscalar mesons, which has been carried out by using GAMS data for $\pi^- p \rightarrow \pi^0 \pi^0 n$ [22], shows that $q\bar{q}$ component in $f_0(980)$ is comparatively compact, $R_{f_0(980)}^2 = 6 \pm 6 \text{ GeV}^{-2}$ [23]. So, basing on GAMS data, it is reasonable to put

$$6 \text{ GeV}^{-2} \leq R_{f_0(980)}^2 \leq 12 \text{ GeV}^{-2}, \quad (2)$$

that favours the large values for the mixing angle.

Concerning the scheme under investigation, it is of principal importance to verify if the transition $\phi(1020) \rightarrow \gamma f_0(980) \rightarrow \gamma \pi^0 \pi^0$ describes the measured $\pi^0 \pi^0$ spectrum within the Flatté formula [24], with parameters determined by hadronic reactions. In Section 3 we calculate the $\pi^0 \pi^0$ spectrum using the parametrization of the Flatté formula suggested in [5,25], and reasonable agreement with data is obtained. The performed spectrum calculations indicate the existence of significant systematic errors in extracting the resonance signal $\phi(1020) \rightarrow \gamma f_0(980)$ which are caused by the background contribution.

Radiative decay $f_0(980) \rightarrow \gamma\gamma$ provides us another source of information about the content of $f_0(980)$: the calculation of partial width $f_0(980) \rightarrow \gamma\gamma$ has been performed in [26] assuming the $q\bar{q}$ structure of $f_0(980)$. However, as was stressed in [27], the extraction of the signal $f_0(980) \rightarrow \gamma\gamma$ from the measured spectra $\gamma\gamma \rightarrow \pi\pi$ meets with a difficulty, that is, a strong interference of the resonance with background, thus giving uncontrollable errors. The recently obtained partial width $\Gamma(f_0(980) \rightarrow \gamma\gamma) = 0.28_{-0.13}^{+0.09} \text{ keV}$ [28] is a factor two less than the averaged value reported previously ($0.56 \pm 0.11 \text{ keV}$ [29]). In Section 4 we re-analyse the decay $f_0(980) \rightarrow \gamma\gamma$ using the Boglione-Pennington number for the width ($0.28_{-0.13}^{+0.09} \text{ keV}$) together with the restriction for the mean radius squared of $f_0(980)$ [23]: $R_{f_0(980)}^2 \leq 12 \text{ GeV}^{-2}$. This provides two possible intervals for the $n\bar{n}/s\bar{s}$ mixing angle:

$$f_0(980) \rightarrow \gamma\gamma: \quad 1) 80^\circ \leq \varphi \leq 93^\circ, \quad 2) (-54^\circ) \leq \varphi \leq (-42^\circ). \quad (3)$$

In Section 5 we calculate partial widths for decays $\phi(1020) \rightarrow \gamma\eta, \gamma\eta', \gamma\pi^0$ in the framework of the same technique as is used for $\phi(1020) \rightarrow \gamma f_0(980)$ and with the same parametrization of the ϕ -meson wave function. The calculations demonstrate a good agreement with the data for these processes as well. The calculation of the decays $\phi(1020) \rightarrow \gamma\eta, \gamma\eta'$ provides us with a strong argument that the applied method for calculating radiative decays of $q\bar{q}$ mesons is wholly reliable, and the decay $\phi(1020) \rightarrow \gamma\pi^0$ allows us to estimate the admixture of the $n\bar{n}$ component in $\phi(1020)$. The partial width of $\phi(1020) \rightarrow \gamma a_0(980)$ is also proportional to the probability of the $n\bar{n}$ component in $\phi(1020)$, we discuss this decay in Section 5 as well.

The restrictions for the mixing angle φ which come from the combined analysis of radiative decays $\phi(1020) \rightarrow \gamma f_0(980)$ and $f_0(980) \rightarrow \gamma\gamma$ are discussed in Section 6. For negative mixing angles the combined analysis does not change the restriction (3):

$$\varphi = -48^\circ \pm 6^\circ, \quad (4)$$

and does not provide any restriction for the radius $R_{f_0(980)}^2(n\bar{n})$.

For positive mixing angles the allowed region of $(\varphi, R_{f_0(980)}^2(n\bar{n}))$ reads as follows:

$$\varphi = 86^\circ \pm 3^\circ, \quad R_{f_0(980)}^2(n\bar{n}) \leq 6.8 \text{ GeV}^{-2}. \quad (5)$$

This means that in this solution $f_0(980)$ should be rather compact $s\bar{s}$ state.

The hadronic decays of $f_0(980)$ strongly contradict the almost complete absence of the $n\bar{n}$ component in $f_0(980)$, that gives the priority to the solution (4), according to which $f_0(980)$ is close to the octet state (recall, $\varphi_{octet} = -54.7^\circ$).

II. $\phi(1020) \rightarrow \gamma f_0(980)$: THE DECAY AMPLITUDE AND PARTIAL WIDTH

In this Section we calculate the coupling constant for the decay $\phi \rightarrow \gamma f_0(980)$ assuming the $q\bar{q}$ structure of $f_0(980)$. In the calculation we use the spectral integration technique developed in [18]. Within the framework of this approach, we study the reaction with a virtual photon, $\phi \rightarrow \gamma(q^2) f_0(980)$, and then the limit $q^2 \rightarrow 0$ is attained.

A. Transition amplitude

The transition amplitude $\phi \rightarrow \gamma(q^2) f_0(980)$ contains S and D waves. Correspondingly, the spin-dependent part of the amplitude, $A_{\mu\nu}$ (the index μ stands for the spin of the photon and ν to that of the ϕ -meson), consists of two terms which are proportional to $g_{\mu\nu}^\perp$ (S wave) and $q_\mu^\perp q_\nu^\perp - q_\perp^2 g_{\mu\nu}^\perp/3$ (D wave):

$$A_{\mu\nu} = e \left[g_{\mu\nu}^\perp F_S(q^2) + (q_\mu^\perp q_\nu^\perp - \frac{1}{3} g_{\mu\nu}^\perp q_\perp^2) F_D(q^2) \right]. \quad (6)$$

Here q^\perp is the relative momentum of the produced particles (it is orthogonal to the ϕ meson momentum p) and $g_{\mu\nu}^\perp$ is a metric tensor in the space orthogonal to p :

$$q_\mu^\perp = q_\mu - p_\mu \frac{(qp)}{p^2}, \quad g_{\mu\nu}^\perp = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}. \quad (7)$$

The requirement $q_\mu A_{\mu\nu} = 0$ results in the following constraint for the invariant amplitudes $F_S(q^2)$ and $F_D(q^2)$:

$$F_S(q^2) + \frac{2}{3}(qq^\perp)F_D(q^2) = 0, \quad (8)$$

hence equation (6) can be re-written as

$$A_{\mu\nu} = \frac{3}{2} e F_S(q^2) \left(g_{\mu\nu}^\perp - \frac{q_\mu^\perp q_\nu^\perp}{q_\perp^2} \right) \equiv e A_{\phi \rightarrow \gamma f_0}(q^2) g_{\mu\nu}^{\perp\perp}. \quad (9)$$

The metric tensor $g_{\mu\nu}^{\perp\perp}$ works in the space orthogonal to the reaction momenta: $g_{\mu\nu}^{\perp\perp} p_\nu = 0$ and $g_{\mu\nu}^{\perp\perp} q_\mu = 0$.

B. Partial width

The decay partial width is determined as

$$m_\phi \Gamma_{\phi \rightarrow f_0 \gamma} = \frac{1}{3} \int d\Phi(p; q, k_f) |A_{\mu\nu}|^2. \quad (10)$$

In (10) the averaging over spin projections of the ϕ -meson and summing over photon ones are carried out; $d\Phi$ is the two-particle phase space which is defined as follows:

$$d\Phi(p; k_1, k_2) = \frac{1}{2} \frac{d^3 k_1}{(2\pi)^3 2k_{10}} \frac{d^3 k_2}{(2\pi)^3 2k_{20}} (2\pi)^4 \delta^{(4)}(p - k_1 - k_2). \quad (11)$$

For the radiative decay $\phi \rightarrow \gamma + f_0(980)$, one has $\int d\Phi(p; q, k_f) = (m_\phi^2 - M_f^2)/(16\pi m_\phi^2)$.

After summing over spin variables, the partial width reads:

$$m_\phi \Gamma_{\phi \rightarrow f_0 \gamma} = \frac{1}{6} \alpha \frac{m_\phi^2 - M_f^2}{m_\phi^2} |A_{\phi \rightarrow \gamma f_0}(0)|^2. \quad (12)$$

Here $\alpha = e^2/4\pi = 1/137$.

C. Double spectral representation for the transition form factor $\phi(1020) \rightarrow \gamma f_0(980)$

Assuming the $q\bar{q}$ structure of $f_0(980)$, the coupling constant of the decay $\phi \rightarrow \gamma(q^2)f_0(980)$ is determined by the processes $\phi \rightarrow q\bar{q}$ and $q\bar{q} \rightarrow f_0(980)$, with the emission of a photon by the quark in the intermediate state, see Fig. 1a. Following the prescriptions of Ref. [18], we calculate the three-point quark loop shown in Fig. 1b by using the double spectral representation over intermediate $q\bar{q}$ states marked by dashed lines in Fig. 1b.

To be illustrative, let us start with the three-point Feynman diagram.

1. Double spectral integral for three-point quark diagram

For the process $V \rightarrow \gamma S$, where V and S stand for scalar and vector particle, the Feynman diagram of Fig. 1a reads:

$$A_{\mu\nu}^{(Feynman)} = \int \frac{d^4k}{i(2\pi)^4} G_V \frac{Z_{V \rightarrow \gamma S}^{(q\bar{q})} S_{\mu\nu}^{(V \rightarrow S)}}{(m^2 - k_1^2)(m^2 - k_1'^2)(m^2 - k_2^2)} G_S. \quad (13)$$

Here k_1, k_1', k_2 are quark momenta, m is the quark mass, and G_V, G_S are quark-meson vertices; the quark charges are included into $Z_{V \rightarrow \gamma S}^{(q\bar{q})}$. The spin-dependent factor reads:

$$S_{\mu\nu}^{(V \rightarrow S)} = -\text{Sp} \left[(\hat{k}_1' + m) \gamma_\mu^\perp (\hat{k}_1 + m) \gamma_\nu^\perp (-\hat{k}_2 + m) \right], \quad (14)$$

where the Dirac matrices γ_μ^\perp and γ_ν^\perp are orthogonal to the emitted momenta: $\gamma_\mu^\perp q_\mu = 0$ and $\gamma_\nu^\perp p_\nu = 0$.

To transform the Feynman integral (13) into the double spectral integral over invariant $q\bar{q}$ masses squared, one should make the following steps:

- (i) consider the corresponding energy-off-shell diagram, Fig. 1b, with $P^2 = (k_1 + k_2)^2 \geq 4m^2$, $P'^2 = (k_1' + k_2)^2 \geq 4m^2$ and fixed momentum transfer squared $q^2 = (P - P')^2$,
- (ii) extract the invariant amplitude separating spin operators,
- (iii) calculate the discontinuities of the invariant amplitude over intermediate $q\bar{q}$ states marked in Fig. 1b by dashed lines.

The double discontinuity is the integrand of the spectral integral over P^2 and P'^2 . Furthermore, we put the following notations:

$$P^2 = s, \quad P'^2 = s'. \quad (15)$$

For the calculation of discontinuity, by cutting Feynman diagram, the pole terms of the propagators are replaced with their residues: $(m^2 - k^2)^{-1} \rightarrow \delta(m^2 - k^2)$. So, the particles in the intermediate states marked by dashed lines I and II in Fig. 1b are mass-on-shell, $k_1^2 = k_2^2 = k_1'^2 = m^2$. As a result, the Feynman diagram integration turns into the integration over phase spaces of the cut diagram states. Corresponding phase space for the three-point diagram reads:

$$d\Phi(P, P'; k_1, k_2, k_1') = d\Phi(P; k_1, k_2) d\Phi(P'; k_1', k_2') (2\pi)^3 2k_{20} \delta^{(3)}(\vec{k}_2' - \vec{k}_2), \quad (16)$$

where the invariant two-particle phase space $d\Phi(P; k_1, k_2)$ is determined as follows:

$$d\Phi(P; k_1, k_2) = \frac{1}{2} \frac{d^3k_1}{(2\pi)^3 2k_{10}} \frac{d^3k_2}{(2\pi)^3 2k_{20}} (2\pi)^4 \delta^{(4)}(P - k_1 - k_2). \quad (17)$$

The last step is to single out the invariant component from the spin factor (14). According to (9), the spin factor (14) is proportional to the metric tensor: $S_{\mu\nu}^{(V \rightarrow S)} \sim g_{\mu\nu}^{\perp\perp}$. Then the spin factor $S_{V \rightarrow \gamma S}^{(tr)}$, determined as

$$S_{\mu\nu}^{(V \rightarrow S)} = g_{\mu\nu}^{\perp\perp} S_{V \rightarrow \gamma S}^{(tr)}(s, s', q^2), \quad (18)$$

is equal to:

$$S_{V \rightarrow \gamma S}^{(tr)}(s, s', q^2) = -2m \left(4m^2 + s - s' - \frac{4ss'}{s + s' - q^2} \alpha(s, s', q^2) \right),$$

$$\alpha(s, s', q^2) = \frac{q^2(s + s' - q^2)}{2q^2(s + s') - (s - s')^2 - q^4}. \quad (19)$$

Here we have used that $(k_1 k_2) = s/2 - m^2$, $(k'_1 k_2) = s'/2 - m^2$ and $(k'_1 k_1) = m^2 - q^2/2$.

The spectral integration is carried out over the energy squared of quarks in the intermediate states, $s = P^2 = (k_1 + k_2)^2$ and $s' = P'^2 = (k'_1 + k_2)^2$, at fixed $q^2 = (P' - P)^2$. The spectral representation for the amplitude $A_{\mu\nu}^{(Feynman)} = g_{\mu\nu}^\perp A_{V \rightarrow \gamma S}^{(Feynman)}(q^2)$ reads:

$$\begin{aligned} A_{V \rightarrow \gamma S}^{(Feynman)}(q^2) &= \int_{4m^2}^{\infty} \frac{ds}{\pi} \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{G_V}{s - M_V^2} \frac{G_S}{s' - M_S^2} \\ &\times \int d\Phi(P, P'; k_1, k'_1, k_2) S_{V \rightarrow \gamma S}^{(tr)}(s, s', q^2) Z_{V \rightarrow \gamma S}^{(q\bar{q})}. \end{aligned} \quad (20)$$

It is reasonable to name the ratios $G_V/(s - M_V^2)$ and $G_S/(s' - M_S^2)$ the wave functions of vector and scalar particle, respectively:

$$\frac{G_V}{s - M_V^2} = \psi_V(s), \quad \frac{G_S}{s' - M_S^2} = \psi_S(s'). \quad (21)$$

Equation (20) is a spectral representation of the Feynman amplitude shown in Fig. 1a for point-like vertices. In general case it is necessary to take account of the s -dependent vertices of the reaction.

2. Form factor $\phi(1020) \rightarrow \gamma f_0(980)$

Generally, the energy-dependent vertices can be incorporated into spectral integrals. According to [18,19], the form factor of a composite system can be obtained by considering the two-particle partial-wave scattering amplitude $1 + 2 \rightarrow 1 + 2$, with the same quantum numbers as for the composite system. In this amplitude, the composite system reveals itself as a pole. The amplitude for the emission of a photon by the two-particle-interaction system has two poles related to the states *before* and *after* electromagnetic interaction, and the two-pole residue of this amplitude provides us form factor of the composite system. When a partial-wave scattering amplitude is treated using the dispersion relation N/D -method, the vertex $G(s)$ is determined by the N -function: the vertex as well as N -function have left-hand-side singularities which are due to forces between particles $1 + 2$.

Generally, the form factor for the transition $\phi(1020) \rightarrow \gamma(q^2) f_0(980)$ reads:

$$\begin{aligned} A_{\phi \rightarrow \gamma f_0}(q^2) &= \int_{4m^2}^{\infty} \frac{ds}{\pi} \int_{4m^2}^{\infty} \frac{ds'}{\pi} \psi_\phi(s) \psi_{f_0}(s') \\ &\times \int d\Phi(P, P'; k_1, k'_1, k_2) S_{\phi \rightarrow \gamma f_0}^{(tr)}(s, s', q^2) Z_{\phi \rightarrow \gamma f_0}^{(q\bar{q})}. \end{aligned} \quad (22)$$

The factor $Z_{\phi \rightarrow f_0}^{(q\bar{q})}$ is defined by the flavor meson component: for $q\bar{q} = n\bar{n}$ we have $Z_{\phi \rightarrow f_0}^{(n\bar{n})} = \sqrt{2}/3$ and for the $s\bar{s}$ component $Z_{\phi \rightarrow f_0}^{(s\bar{s})} = -2/3$.

Working with Eqs. (22), one can either

- (i) express it in terms of the light cone variables, or
- (ii) keep the spectral integrals over s and s' and eliminate integrations over quark momenta with the help of the phase space δ -functions.

a) Light-cone variables

The formulae (16), (17), (22) allow one to make the transformation to the light-cone variables, using the boost along the z -axis. Let us use the frame where the initial vector meson moves along the z -axis with the momentum $p \rightarrow \infty$:

$$P = (p + \frac{s}{2p}, 0, p), \quad P' = (p + \frac{s' + q_\perp^2}{2p}, -\vec{q}_\perp, p). \quad (23)$$

In this frame the two-particle phase space is equal to

$$d\Phi(P; k_1, k_2) = \frac{1}{16\pi^2} \frac{dx_1 dx_2}{x_1 x_2} d^2 k_{1\perp} d^2 k_{2\perp} \delta(1 - x_1 - x_2) \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp}) \\ \times \delta\left(s - \frac{m^2 + k_{1\perp}^2}{x_1} - \frac{m^2 + k_{2\perp}^2}{x_2}\right), \quad (24)$$

and the phase space for the triangle diagram reads:

$$d\Phi(P, P'; k_1, k_2, k'_1) = \frac{1}{16\pi} \frac{dx_1 dx_2}{x_1^2 x_2} d^2 k_{1\perp} d^2 k_{2\perp} \delta(1 - x_1 - x_2) \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp}) \\ \times \delta\left(s - \frac{m^2 + k_{1\perp}^2}{x_1} - \frac{m^2 + k_{2\perp}^2}{x_2}\right) \\ \times \delta\left(s' + q_\perp^2 - \frac{m^2 + (\vec{k}_{1\perp} - \vec{q}_\perp)^2}{x_1} - \frac{m^2 + k_{2\perp}^2}{x_2}\right). \quad (25)$$

After having integrated over s and s' , by using δ -functions, we have:

$$A_{\phi \rightarrow \gamma(q^2)f_0}(q^2) = \frac{Z_{\phi \rightarrow \gamma f_0}^{(q\bar{q})}}{16\pi^3} \int_0^1 \frac{dx}{x(1-x)^2} \int d^2 k_\perp \psi_\phi(s) \psi_{f_0}(s') S_{\phi \rightarrow \gamma f_0}^{(tr)}(s, s', q^2), \quad (26)$$

where $x = k_{2z}/p$, $\vec{k}_\perp = \vec{k}_{2\perp}$, and the $q\bar{q}$ invariant energies squared are

$$s = \frac{m^2 + k_\perp^2}{x(1-x)}, \quad s' = \frac{m^2 + (\vec{k}_\perp + x\vec{q}_\perp)^2}{x(1-x)}. \quad (27)$$

b) Spectral integral representation

In Eq. (22) one can integrate over the phase space keeping fixed the energies squared, s and s' . After using the phase space δ -functions, we have:

$$A_{\phi \rightarrow \gamma f_0}(q^2) = \int_{4m^2}^\infty \frac{ds ds'}{\pi^2} \psi_\phi(s) \psi_{f_0}(s') \frac{\theta(ss'Q^2 - m^2 \lambda(s, s', Q^2))}{16\sqrt{\lambda(s, s', Q^2)}} \\ \times Z_{V \rightarrow \gamma S}^{(q\bar{q})} S_{V \rightarrow \gamma S}^{(tr)}(s, s', Q^2), \quad (28)$$

with

$$\lambda(s, s', Q^2) = (s' - s)^2 + 2Q^2(s' + s) + Q^4. \quad (29)$$

The θ -function, $\theta(X)$, restricts the integration region for different $Q^2 = -q^2$: $\theta(X) = 1$ at $X \geq 0$ and $\theta(X) = 0$ at $X < 0$.

In the limit $Q^2 \rightarrow 0$, the integration over s' is carried out, and we have:

$$A_{\phi \rightarrow \gamma f_0}(0) = \int_{4m^2}^\infty \frac{ds}{\pi} \psi_\phi(s) \psi_{f_0}(s) \left[-\frac{m^3}{2\pi} \ln \frac{\sqrt{s} + \sqrt{s - 4m^2}}{\sqrt{s} - \sqrt{s - 4m^2}} + \frac{m}{4\pi} \sqrt{s(s - 4m^2)} \right] Z_{\phi \rightarrow \gamma f_0}^{(q\bar{q})}. \quad (30)$$

To calculate $A_{\phi \rightarrow \gamma f_0}(0)$ one should determine the wave functions of the ϕ -meson and $f_0(980)$.

3. Wave functions of $\phi(1020)$ and $f_0(980)$.

We write down the wave functions of $\phi(1020)$ and $f_0(980)$ expanding them in a series as follows:

$$\Psi_\phi(s) = (n\bar{n} \sin \varphi_V + s\bar{s} \cos \varphi_V) \psi_\phi(s), \\ \Psi_f(s) = (n\bar{n} \cos \varphi + s\bar{s} \sin \varphi) \psi_f(s). \quad (31)$$

For $\psi_\phi(s)$ and $\psi_f(s)$ the exponential parametrization is used:

$$\psi_\phi(s) = C_\phi e^{-b_\phi s}, \quad \psi_f(s) = C_f e^{-b_f s}. \quad (32)$$

The parameters b_ϕ and b_f characterise the size of the system, they are related to the mean radii squared of these mesons, R_ϕ^2 and $R_{f_0(980)}^2$ and C_ϕ and C_f are the normalization constants. In our approach the dynamics of interaction is hidden in the wave functions ψ_ϕ and ψ_{f_0} . We assume exponential s -dependence for the wave functions. Recall that exponential parametrization is often used in the quark model. Working upon meson radiative decays, we have additional arguments to employ this presentation of wave functions of the low-lying $q\bar{q}$ states.

In Ref. [18], in our analysis of pion form factor as well as transition form factors $\pi^0 \rightarrow \gamma\gamma^*(Q^2)$, $\eta \rightarrow \gamma\gamma^*(Q^2)$ and $\eta' \rightarrow \gamma\gamma^*(Q^2)$, we have reconstructed wave functions of pseudoscalar mesons as functions of s . At small relative momenta of quark and antiquark, $k^2 \leq 0.6$ (GeV/c) 2 (where $k^2 = s/4 - m^2$), the wave functions demonstrate rapid – exponential-type – decrease, then it becomes slower. The same behaviour is appropriate to wave function of the photon, which was used in our calculations; it is shown below, in Fig. 5). We keep in mind just this form of the wave functions for low-lying $q\bar{q}$ mesons, that is a foundation for the formula (32). In [30], at calculation of the transitions $\eta \rightarrow \gamma\gamma$, $\eta' \rightarrow \gamma\gamma$, $\eta(1295) \rightarrow \gamma\gamma$ and $\eta(1440) \rightarrow \gamma\gamma$ we checked the possibility to use exponential-type wave functions. The processes $\eta \rightarrow \gamma\gamma$ and $\eta' \rightarrow \gamma\gamma$ have been calculated both with the wave function reconstructed from the experiment and exponential-type wave functions, with the same R^2 . The difference between form factors is not greater than 5%.

At fixed R_ϕ^2 and $R_{f_0(980)}^2$ the constants C_ϕ and C_f are determined by the wave function normalization, which itself is given by the meson form factor in the external field, $F_{meson}(q^2)$, namely, at small q^2 :

$$F_{meson}(q^2) \simeq 1 + \frac{1}{6} R_{meson}^2 q^2. \quad (33)$$

The requirement $F_{meson}(0) = 1$ fixes the constant C_{meson} in (32), while the value R_{meson}^2 is directly related to the parameter b_{meson} .

The form factor $F_{meson}(q^2)$ reads [18]:

$$F_{meson}(q^2) = \int_{4m^2}^{\infty} \frac{ds ds'}{\pi^2} \psi_{meson}(s) \psi_{meson}(s') \frac{\theta(ss'Q^2 - m^2 \lambda(s, s', Q^2))}{16 \sqrt{\lambda(s, s', Q^2)}} S_{meson}^{(tr)}(s, s', q^2), \quad (34)$$

where $S_{meson}^{(tr)}$ for $f_0(980)$ and $\phi(1020)$ is determined by the following traces:

$$\begin{aligned} 2P_\mu^\perp S_{f_0}^{(tr)}(s, s', q^2) &= -\text{Sp} \left[(\hat{k}_1' + m) \gamma_\mu^\perp (\hat{k}_1 + m) (-\hat{k}_2 + m) \right], \\ 2P_\mu^\perp S_\phi^{(tr)}(s, s', q^2) &= -\frac{1}{3} \text{Sp} \left[\gamma_\alpha'^\perp (\hat{k}_1' + m) \gamma_\mu^\perp (\hat{k}_1 + m) \gamma_\alpha^\perp (-\hat{k}_2 + m) \right]. \end{aligned} \quad (35)$$

Here the orthogonal components are as follows:

$$\begin{aligned} P_\mu^\perp &= P_\mu - q_\mu \frac{(Pq)}{q^2}, \quad \gamma_\mu^\perp = \gamma_\mu - q_\mu \frac{\hat{q}}{q^2}, \\ \gamma_\alpha^\perp &= \gamma_\alpha - P_\alpha \frac{\hat{P}}{P^2}, \quad \gamma_\alpha'^\perp = \gamma_\alpha - P_\alpha' \frac{\hat{P}'}{P'^2}, \end{aligned} \quad (36)$$

and $q = k_1' - k_1$. When determining $S_\phi^{(tr)}$, we have averaged over polarizations of the ϕ meson (the factor 1/3).

The functions $S_{f_0}^{(tr)}$ and $S_\phi^{(tr)}$ are equal to:

$$\begin{aligned} S_{f_0}^{(tr)}(s, s', q^2) &= \alpha(s, s', q^2)(s + s' - 8m^2 - q^2) + q^2, \\ S_\phi^{(tr)}(s, s', q^2) &= \frac{2}{3} [\alpha(s, s', q^2)(s + s' + 4m^2 - q^2) + q^2]. \end{aligned} \quad (37)$$

The radius squared of the $n\bar{n}$ component in the ϕ -meson is suggested to be approximately the same as that of the pion: $R_\phi^2(n\bar{n}) \simeq 10.9$ GeV $^{-2}$, while the radius squared $R_\phi^2(s\bar{s})$ is slightly less, $R_\phi^2(s\bar{s}) \simeq 9.3$ GeV $^{-2}$ (that corresponds to $b_\phi = 2.5$ GeV $^{-2}$). As to $f_0(980)$, we vary its radius in the interval 6 GeV $^{-2} \leq R_{f_0(980)}^2 \leq 18$ GeV $^{-2}$.

4. The result for $\phi(1020) \rightarrow \gamma + f_0(980)$.

The amplitude $A_{\phi \rightarrow \gamma f_0}(0)$ is a sum of two terms related to the $n\bar{n}$ and $s\bar{s}$ components:

$$A_{\phi \rightarrow \gamma f_0}(0) = \cos \varphi \sin \varphi_V A_{\phi \rightarrow \gamma f_0}^{(n\bar{n})}(0) + \sin \varphi \cos \varphi_V A_{\phi \rightarrow \gamma f_0}^{(s\bar{s})}(0) . \quad (38)$$

In our estimations we put $\cos \varphi_V \sim 0.99$ and, correspondingly, $|\sin \varphi_V| \leq 0.1$; for $f_0(980)$ we vary the mixing angle in the interval $0^\circ \leq |\varphi| \leq 90^\circ$.

The results of the calculation are shown in Figs. 2 and 3. In Fig. 2 the values $A_{\phi \rightarrow \gamma f_0}^{(n\bar{n})}(0)$ and $A_{\phi \rightarrow \gamma f_0}^{(s\bar{s})}(0)$ are plotted with respect to the radius squared $R_{f_0(980)}^2$, while the mean radius squared of the ϕ -meson is close to that of the pion: $R_{\phi(1020)}^2 \simeq 0.4 \text{ fm}^2$.

In Fig. 3 one can see the value $BR(\phi \rightarrow \gamma f_0(980))$ at various $\sin |\varphi|$: $\sin |\varphi| = 0.4, 0.6, 0.8, 0.9, 0.985$. Shaded areas correspond to the variation of φ_V in the interval $-8^\circ \leq \varphi_V \leq 8^\circ$; the lower and upper curves of the shaded area correspond to destructive and constructive interferences of $A_{\phi \rightarrow \gamma f_0}^{(n\bar{n})}(0)$ and $A_{\phi \rightarrow \gamma f_0}^{(s\bar{s})}(0)$, respectively.

The measurement of the $f_0(980)$ signal in the $\gamma\pi^0\pi^0$ reaction gives the branching ratio $BR(\phi \rightarrow \gamma f_0(980)) = (3.5 \pm 0.3^{+1.3}_{-0.5}) \cdot 10^{-4}$ [8], in the analysis of $\gamma\pi^0\pi^0$ and $\gamma\pi^+\pi^-$ channels it was found $BR(\phi \rightarrow \gamma f_0(980)) = (2.90 \pm 0.21 \pm 1.5) \cdot 10^{-4}$ [7]; the averaged value is given in [16]: $BR(\phi \rightarrow \gamma f_0(980)) = (3.4 \pm 0.4) \cdot 10^{-4}$. Our calculations of the $\pi^0\pi^0$ spectrum with the conventional Flatté formula (Section 4) support the statement of [7,8] about the presence of large systematic errors related to the procedure of extracting the $f_0(980)$ signal. In our estimation of the permissible interval for the mixing angle φ , we have used the averaged value given by [16], with the inclusion of systematic errors of the order of those found in [7,8]: $BR(\phi \rightarrow \gamma f_0(980)) = (3.4 \pm 0.4^{+1.5}_{-0.5}) \cdot 10^{-4}$.

The calculated values of $BR(\phi \rightarrow \gamma f_0(980))$ agree with experimental data for $|\varphi| \geq 25^\circ$; larger values of the mixing angle, $|\varphi| \geq 55^\circ$, correspond to a more compact structure of $f_0(980)$, namely, $R_{f_0(980)}^2 \leq 10 \text{ GeV}^{-2}$, while small mixing angles $|\varphi| \sim 25^\circ$ are related to a loosely bound structure, $R_{f_0(980)}^2 \geq 12 \text{ GeV}^{-2}$ (recall that for the pion $R_\pi^2 \simeq 10 \text{ GeV}^{-2}$).

The evaluation of the radius of $f_0(980)$ was performed in [26] on the basis of GAMS data [22], where the t -dependence was measured in the process $\pi^-p \rightarrow f_0(980)n$: these data favour the comparatively compact structure of the $q\bar{q}$ component in $f_0(980)$, namely, $R_{f_0(980)}^2 = 6 \pm 6 \text{ GeV}^{-2}$.

III. CALCULATION OF THE PARTIAL WIDTH FOR $\phi(1020) \rightarrow \gamma\pi^0\pi^0$

In the $\pi^0\pi^0$ spectrum, the resonance $f_0(980)$ is seen as a peak on the edge of the spectrum. So it is rather instructive to calculate the dependence of $BR(\phi(1020) \rightarrow \gamma\pi^0\pi^0)$ on $M_{\pi\pi}$ by using the conventional Flatté formula [24].

The resonance $f_0(980)$ in hadronic reactions is well described by the Flatté formula which takes into account two decay channels, $f_0(980) \rightarrow \pi\pi$ and $f_0(980) \rightarrow K\bar{K}$; the parameters of the Flatté formula may be found in [5,25].

In the calculation of the $\pi\pi$ spectrum, we take into account the resonance contribution and the background, $B(M_{\pi\pi}^2)$. The partial width $d\Gamma_{\phi \rightarrow \gamma\pi\pi}$ reads:

$$d\Gamma_{\phi \rightarrow \gamma\pi\pi} = \Gamma_{\phi \rightarrow \gamma f_0(980)} \frac{M_f(m_\phi^2 - M_{\pi\pi}^2)}{M_{\pi\pi}(m_\phi^2 - M_f^2)} \frac{F_{thresh}^2(M_{\pi\pi}^2)}{F_{thresh}^2(M_f^2)} \times \left| \frac{1}{M_0^2 - M_{\pi\pi}^2 - ig_\pi^2 \rho_{\pi\pi} - ig_K^2 \rho_{K\bar{K}}} + B(M_{\pi\pi}^2) \right|^2 g_\pi^2 \rho_{\pi\pi} \frac{dM_{\pi\pi}^2}{\pi} , \quad (39)$$

where $M_f = 0.98 \text{ GeV}$. $F_{thresh}(M_{\pi\pi}^2)$ is a factor which provides the threshold behaviour of the decay amplitude $\phi(1020) \rightarrow \gamma\pi^0\pi^0$ at $M_{\pi\pi}^2 \rightarrow m_\phi^2$. We have chosen it as $F_{thresh}^2(M_{\pi\pi}^2) = 1 - \exp[-(M_{\pi\pi}^2 - m_\phi^2)^2/\mu_0^4]$, where μ_0 is a scale parameter; below we put $\mu_0 = 2m_\pi$.

Here we separate $\Gamma_{\phi \rightarrow \gamma f_0(980)}$ and introduce the ratio of phase spaces for $(\pi\pi) + \gamma$ and $f_0(980) + \gamma$:

$$\frac{d\Phi_{(\pi\pi)\gamma}}{d\Phi_{f_0(980)\gamma}} = \frac{M_f(m_\phi^2 - M_{\pi\pi}^2)}{M_{\pi\pi}(m_\phi^2 - M_f^2)} . \quad (40)$$

The values g_π and g_K are coupling constants for the transitions $f_0(980) \rightarrow \pi\pi, K\bar{K}$. Also $\rho_{\pi\pi}$ and $\rho_{K\bar{K}}$ are phase spaces for the $\pi\pi$ and $K\bar{K}$ states: $\rho_{\pi\pi} = \sqrt{M_{\pi\pi}^2 - 4m_\pi^2}/(16\pi M_{\pi\pi})$ and $\rho_{K\bar{K}} = \sqrt{M_{\pi\pi}^2 - 4m_K^2}/(16\pi M_{\pi\pi})$; recall that in the case of $M_{\pi\pi}^2 < 4m_K^2$ the kaon phase space is imaginary, for $\sqrt{M_{\pi\pi}^2 - 4m_K^2} \rightarrow i\sqrt{4m_K^2 - M_{\pi\pi}^2}$.

According to [5,25], the Flatté formula is parametrized as follows:

$$g_\pi^2 \rho_{\pi\pi} \rightarrow (0.12 \text{ GeV}) \sqrt{M_{\pi\pi}^2 - 4m_\pi^2}, \quad g_K^2 \rho_{K\bar{K}} \rightarrow (0.27 \text{ GeV}) \sqrt{M_{\pi\pi}^2 - 4m_K^2}, \quad (41)$$

$$M_0 = 0.99 \pm 0.01 \text{ GeV}.$$

We parametrize the background term as $B(M_{\pi\pi}^2) = a + bM_{\pi\pi}^2$ where a and b are complex constants. The unitarity condition in the $\pi\pi$ channel tells us that the decay amplitude $\phi(1020) \rightarrow \gamma\pi\pi$ has the complex phase factor related to the $\pi\pi$ phase shift in the $(IJ^{PC} = 00^{++})$ channel: $A_{\gamma\pi\pi} = |A_{\gamma\pi\pi}| \exp[i\delta_0^0(M_{\pi\pi})]$. Therefore, one has

$$\begin{aligned} & \left| \frac{1}{M_0^2 - M_{\pi\pi}^2 - ig_\pi^2 \rho_{\pi\pi} - ig_K^2 \rho_{K\bar{K}}} + B(M_{\pi\pi}^2) \right| e^{i\delta_0^0(M_{\pi\pi})} \\ &= \left(\frac{1}{M_0^2 - M_{\pi\pi}^2 - ig_\pi^2 \rho_{\pi\pi} - ig_K^2 \rho_{K\bar{K}}} + B(M_{\pi\pi}^2) \right) e^{i\Delta\delta_0^0(M_{\pi\pi})}. \end{aligned} \quad (42)$$

We parametrize the phase $\Delta\delta_0^0(M_{\pi\pi})$ as $\Delta\delta_0^0(M_{\pi\pi}) = \Delta_0 + \Delta_1(M_{\pi\pi}/m_0 - 1) + \Delta_2(M_{\pi\pi}/m_0 - 1)^2$ with $m_0 = 1 \text{ GeV}$. The complexity of $B(M_{\pi\pi}^2)$ is determined by the difference of phases in the terms for the $f_0(980)$ production and primary background contribution.

To calculate the $\gamma\pi^0\pi^0$ spectrum, one should multiply (39) by the factor related to the $\pi^0\pi^0$ channel $d\Gamma_{\phi \rightarrow \gamma\pi^0\pi^0} = \frac{1}{3} d\Gamma_{\phi \rightarrow \gamma\pi\pi}$. Figure 4 demonstrates $BR(\phi \rightarrow \gamma\pi^0\pi^0)$ calculated using Eq. (39), with $BR(\phi \rightarrow \gamma f_0(980)) = 3.4 \cdot 10^{-4}$, $5.4 \cdot 10^{-4}$ and $M_0 = 0.98 \text{ GeV}$, 1.00 GeV . The following parameters (in GeV units) are used for the background term $B(M_{\pi\pi}^2)$ and the phase $\Delta\delta_0^0(M_{\pi\pi})$:

$$\begin{aligned} (a) \text{ BR} = 3.4 \times 10^{-4}, \quad M_0 = 0.98 : \quad & a = -1.24 + i0.74, \quad b = 1.91 - i0.95, \\ & \Delta_0 = 99^\circ, \Delta_1 = 11^\circ, \Delta_2 = 440^\circ; \\ (b) \text{ BR} = 3.4 \times 10^{-4}, \quad M_0 = 1.00 : \quad & a = -1.96 + i0.073, \quad b = 2.37 + i1.81, \\ & \Delta_0 = 106^\circ, \Delta_1 = -136^\circ, \Delta_2 = 341^\circ; \\ (c) \text{ BR} = 5.4 \times 10^{-4}, \quad M_0 = 0.98 : \quad & a = -0.84 + i0.91, \quad b = 0.73 - i2.63, \\ & \Delta_0 = 98^\circ, \Delta_1 = 203^\circ, \Delta_2 = 735^\circ; \\ (d) \text{ BR} = 5.4 \times 10^{-4}, \quad M_0 = 1.00 : \quad & a = -1.13 + i0.69, \quad b = 1.39 - i1.13, \\ & \Delta_0 = 94^\circ, \Delta_1 = 9^\circ, \Delta_2 = 462^\circ. \end{aligned} \quad (43)$$

It is seen that the data [8] agree reasonably with the Flatté parametrization for hadronic reactions given by [5,25]. A good description of the $\pi^0\pi^0$ spectrum at $BR(\phi \rightarrow \gamma f_0(980)) = 5.4 \cdot 10^{-4}$ supports the statement of [7,8] about large systematic errors in the determination of the $\gamma f_0(980)$ signal.

IV. RADIATIVE DECAY $f_0(980) \rightarrow \gamma\gamma$

The amplitude for the transition $f_0(980) \rightarrow \gamma(q^2)\gamma(q'^2)$ has the same spin structure as $\phi \rightarrow \gamma f_0(980)$:

$$A_{\mu\nu} = e^2 A_{f_0 \rightarrow \gamma\gamma}(q^2) g_{\mu\nu}^{\perp\perp} \quad (44)$$

where $g_{\mu\nu}^{\perp\perp} q'_\nu = 0$ and $g_{\mu\nu}^{\perp\perp} q_\mu = 0$.

The invariant amplitude reads:

$$A_{f_0 \rightarrow \gamma\gamma}(q^2) = \sqrt{N_c} \int_{4m^2}^{\infty} \frac{ds ds'}{\pi^2} \psi_{f_0}(s) \psi_\gamma(s') \frac{\theta(ss'Q^2 - m^2 \lambda(s, s', Q^2))}{16\sqrt{\lambda(s, s', Q^2)}} Z_{f_0 \rightarrow \gamma\gamma}^{(q\bar{q})} S_{f_0 \rightarrow \gamma\gamma}^{(tr)}(s, s', Q^2), \quad (45)$$

where $N_c = 3$ is number of colours, $Z_{f_0 \rightarrow \gamma\gamma}^{(n\bar{n})} = 5\sqrt{2}/9$ and $Z_{f_0 \rightarrow \gamma\gamma}^{(s\bar{s})} = 2/9$. The spin factor $S_{f_0 \rightarrow \gamma\gamma}^{(tr)}(s, s', Q^2)$ is similar to that for $\phi \rightarrow \gamma f_0$, namely:

$$S_{f_0 \rightarrow \gamma\gamma}^{(tr)}(s, s', q^2) = -2m \left(-s + s' + 4m^2 - \frac{4ss'}{s + s' - q^2} \alpha(s, s', q^2) \right). \quad (46)$$

In the limit $Q^2 \rightarrow 0$, the term which makes different $S_{f_0 \rightarrow \gamma\gamma}^{(tr)}(s, s', q^2)$ and $S_{\phi \rightarrow \gamma f_0}^{(tr)}(s, s', q^2)$ vanishes in the integral (45), and we have for $A_{f_0 \rightarrow \gamma\gamma}(0)$ an expression similar to (28):

$$A_{f_0 \rightarrow \gamma\gamma}(0) = \frac{m\sqrt{N_c}Z_{f_0 \rightarrow \gamma\gamma}^{(q\bar{q})}}{4\pi} \int_{4m^2}^{\infty} \frac{ds}{\pi} \psi_{f_0}(s) \psi_{\gamma}(s) \left[\sqrt{s(s-4m^2)} - 2m^2 \ln \frac{\sqrt{s} + \sqrt{s-4m^2}}{\sqrt{s} - \sqrt{s-4m^2}} \right]. \quad (47)$$

The photon wave function was found in the analysis of the transition form factors $\pi^0 \rightarrow \gamma(Q^2)\gamma$, $\eta \rightarrow \gamma(Q^2)\gamma$, and $\eta' \rightarrow \gamma(Q^2)\gamma$ [18]: it is shown in Fig. 5. With this wave function we calculate $A_{f_0 \rightarrow \gamma\gamma}^{n\bar{n}}(0)$ and $A_{f_0 \rightarrow \gamma\gamma}^{s\bar{s}}(0)$; these amplitudes plotted versus $R_{f_0(980)}^2$ are shown in Fig. 2b.

The partial width $\Gamma_{f_0 \rightarrow \gamma\gamma}$ is equal to:

$$M_f \Gamma_{f_0 \rightarrow \gamma\gamma} = \pi \alpha^2 |A_{f_0 \rightarrow \gamma\gamma}(0)|^2, \quad (48)$$

where

$$A_{f_0 \rightarrow \gamma\gamma}(0) = \cos \varphi A_{f_0 \rightarrow \gamma\gamma}^{(n\bar{n})}(0) + \sin \varphi A_{f_0 \rightarrow \gamma\gamma}^{(s\bar{s})}(0). \quad (49)$$

Figure 6 demonstrates the comparison of the calculated $\Gamma_{f_0 \rightarrow \gamma\gamma}$, at different $R_{f_0(980)}^2$ and φ , with the data [28]. It is possible to describe the experimental data ($\Gamma_{f_0(980) \rightarrow \gamma\gamma} = 0.28_{-0.13}^{+0.09}$ [28]) at positive mixing angles as well as at negative ones: $77^\circ \leq \varphi \leq 93^\circ$ and $(-54^\circ) \leq \varphi \leq (-38^\circ)$. The use of the radius restriction (2) makes the allowed interval of mixing angles slightly narrower, see (3).

V. RADIATIVE DECAYS $\phi(1020) \rightarrow \gamma\eta, \gamma\eta', \gamma\pi^0, \gamma a_0(980)$

The decays $\phi(1020) \rightarrow \gamma\eta, \gamma\eta', \gamma\pi^0, \gamma a_0(980)$ do not provide us with direct information on the quark content of $f_0(980)$; still, it looks necessary to perform calculations and comparison with data in order to check the reliability of the method. In addition, the decay $\phi(1020) \rightarrow \gamma\pi^0$ allows us to evaluate the admixture of the $n\bar{n}$ component in the ϕ meson; as we saw in Section 2, this admixture affects significantly the value $\Gamma_{\phi \rightarrow \gamma f_0(980)}$.

The amplitude for the transition $\phi \rightarrow \gamma P$, where $P = \eta, \eta', \pi^0$, has the following structure:

$$A_{\mu\nu} = e A_{\phi \rightarrow \gamma P}(q^2) \epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta. \quad (50)$$

Radiative decay amplitudes $\phi \rightarrow \gamma\eta, \gamma\eta', \gamma\pi^0$ may be calculated similarly to the decay amplitude $\phi \rightarrow \gamma f_0(980)$, see Eq. (28), with necessary substitutions of the wave functions $\psi_{f_0} \rightarrow \psi_\eta, \psi_{\eta'}, \psi_\pi$ as well as charge and spin factors:

$$A_{\phi \rightarrow \gamma P}(0) = \frac{m Z_{\phi \rightarrow \gamma P}^{(q\bar{q})}}{4\pi} \int_{4m^2}^{\infty} \frac{ds}{\pi} \psi_\phi(s) \psi_P(s) \ln \frac{\sqrt{s} + \sqrt{s-4m^2}}{\sqrt{s} - \sqrt{s-4m^2}}. \quad (51)$$

The charge factors for the considered radiative decays are as follows: for the $s\bar{s}$ component in the reactions $\phi \rightarrow \gamma\eta, \gamma\eta'$, $Z_{\phi \rightarrow \gamma\eta}^{(s\bar{s})} = Z_{\phi \rightarrow \gamma\eta'}^{(s\bar{s})} = -2/3$, and for the π^0 production, $Z_{\phi \rightarrow \gamma\pi^0} = 1$.

For the transitions $\phi \rightarrow \gamma\eta$ and $\phi \rightarrow \gamma\eta'$ we take into account the dominant $s\bar{s}$ component only: $-\sin\theta s\bar{s}$ in η -meson and $\cos\theta s\bar{s}$ in η' -meson, and $\sin\theta = 0.6$.

The spin factors are equal to

$$S_{\phi \rightarrow \eta}^{(tr)}(s, s', q^2) = S_{\phi \rightarrow \eta'}(s, s', q^2) = 4m_s, \quad S_{\phi \rightarrow \pi}^{(tr)}(s, s', q^2) = 4m. \quad (52)$$

For pion wave function we have chosen $b_\pi = 2.0 \text{ GeV}^{-2}$ that corresponds to $R_\pi^2 = 10.1 \text{ GeV}^{-2}$, the same radius is fixed for the $n\bar{n}$ component in η and η' . As to the strange component in η and η' , its slope is the same: $b_{\eta(s\bar{s})} = 2 \text{ GeV}^{-2}$ that leads to a smaller radius, $R^2(s\bar{s}) = 8.3 \text{ GeV}^{-2}$.

The results of the calculations versus branching ratios given by PDG-compilation [16] are as follows:

$$\begin{aligned} BR(\phi \rightarrow \gamma\eta) &= 1.46 \cdot 10^{-2}, & BR_{PDG}(\phi \rightarrow \gamma\eta) &= (1.30 \pm 0.03) \cdot 10^{-2}, \\ BR(\phi \rightarrow \gamma\eta') &= 0.97 \cdot 10^{-4}, & BR_{PDG}(\phi \rightarrow \gamma\eta') &= (0.67_{-0.31}^{+0.35}) \cdot 10^{-4}. \end{aligned} \quad (53)$$

The calculated branching ratios agree reasonably with those given in [16].

For the process $\phi \rightarrow \gamma\pi^0$ the compilation [16] gives $BR(\phi \rightarrow \gamma\pi^0) = (1.26 \pm 0.10) \cdot 10^{-3}$, and this value requires $|\sin \varphi_V| \simeq 0.07$ (or $|\varphi_V| \simeq 4^\circ$), for with just this admixture of the $n\bar{n}$ component in $\phi(1020)$ we reach the agreement with data. However, in the estimation of the allowed regions for the mixing angle φ (see Fig. 3), we use

$$|\varphi_V| = 4^\circ \pm 4^\circ \quad (54)$$

considering the accuracy inherent to the quark model to be comparable with the obtained small value of $|\varphi_V|$.

The process $\phi(1020) \rightarrow \gamma a_0(980)$ depends also on the mixing angle $|\varphi_V|$: the decay amplitude is proportional to $\sin \varphi_V$, namely, $A_{\phi \rightarrow \gamma a_0} = \sin \varphi_V A_{\phi \rightarrow \gamma a_0}^{n\bar{n}}$. The amplitude $A_{\phi \rightarrow \gamma a_0}^{n\bar{n}}$ is equal to that for the process $\phi \rightarrow \gamma f_0$, up to a numerical factor: $A_{\phi \rightarrow \gamma a_0}^{n\bar{n}}/A_{\phi \rightarrow \gamma f_0}^{n\bar{n}} = Z_{\phi \rightarrow \gamma a_0}^{n\bar{n}}/Z_{\phi \rightarrow \gamma f_0}^{n\bar{n}} = 3$ because $Z_{\phi \rightarrow \gamma a_0}^{n\bar{n}} = 1$ (the value $3A_{\phi \rightarrow \gamma f_0}^{n\bar{n}}$ is shown on Fig. 2a). We have for the region $R_{a_0(980)}^2 \sim 8 \text{ GeV}^{-2} - 12 \text{ GeV}^{-2}$:

$$BR(\phi(1020) \rightarrow \gamma a_0(980)) \simeq \sin^2 \varphi_V \cdot (14 \pm 3) \cdot 10^{-4} \quad (55)$$

with lower values for $R_{a_0(980)}^2 \sim 8 \text{ GeV}^{-2}$ and larger ones for $R_{a_0(980)}^2 \sim 12 \text{ GeV}^{-2}$. At $\sin^2 \varphi_V \leq 0.02$, we predict $BR(\phi(1020) \rightarrow \gamma a_0(980)) \leq 0.28 \cdot 10^{-4}$.

In recent paper [31] the partial width for $\phi(1020) \rightarrow \gamma\eta\pi^0$ was measured, it was found $BR(\phi(1020) \rightarrow \gamma\eta\pi^0; M_{\eta\pi} > 900 \text{ MeV}) = (0.46 \pm 0.13) \cdot 10^{-4}$. If at $M_{\eta\pi} > 900 \text{ MeV}$ the ratio of the resonance/background is of the order of the unity, that is quite possible, the value found in [31] is in agreement with small value of $|\varphi_V|$.

VI. CONCLUSION

Figure 7 demonstrates the $(\varphi, R_{f_0(980)}^2)$ -plot where the allowed areas for the reactions $\phi(1020) \rightarrow \gamma f_0(980)$ and $\phi(1020) \rightarrow \gamma\gamma$ are shown. We see that the radiative decays $\phi(1020) \rightarrow \gamma f_0(980)$ and $f_0(980) \rightarrow \gamma\gamma$ are well described in the framework of the hypothesis of the dominant $q\bar{q}$ structure of $f_0(980)$. For flavour wave function determined by $\Psi_{flavour}(f_0(980)) = n\bar{n} \cos \varphi + s\bar{s} \sin \varphi$, the solutions found are either $\varphi = -48^\circ \pm 6^\circ$ or $\varphi = 85^\circ \pm 5^\circ$. The solution with negative φ seems more preferable, that means the $q\bar{q}$ component is rather close to the flavour octet ($\varphi_{octet} = -54.7^\circ$). Such a proximity to the octet may be related to the following scenario: the broad resonance $f_0(1530_{250}^{+90})$, which is, according to the K -matrix analysis, the descendant of the lightest scalar glueball after accumulation of widths of neighbouring resonances took the singlet component from a predecessor of $f_0(980)$, thus turning $f_0(980)$ into an almost pure octet.

The dominance of $q\bar{q}$ component does not exclude the existence of the other components in $f_0(980)$. The location of the resonance pole near the $K\bar{K}$ threshold definitely points on a certain admixture of the long-range $K\bar{K}$ component in $f_0(980)$. To investigate this admixture the precise measurements of the $K\bar{K}$ spectra in the interval $1000 - 1150 \text{ MeV}$ are necessary: only these spectra could shed the light on the role of the long-range $K\bar{K}$ component in $f_0(980)$.

The existence of the long-range $K\bar{K}$ component or that of gluonium in the $f_0(980)$ results in a decrease of the $s\bar{s}$ fraction in the $q\bar{q}$ component: for example, if the long-range $K\bar{K}$ (or gluonium) admixture is of the order of 15%, the data require either $\varphi = -45^\circ \pm 6^\circ$ or $\varphi = 83^\circ \pm 4^\circ$.

We thank D. V. Bugg, L. G. Dakhno, V. S. Fadin, L. Montanet and A. V. Sarantsev for useful discussions and illuminative information. The work is supported by the RFBR grant 01-02-17861.

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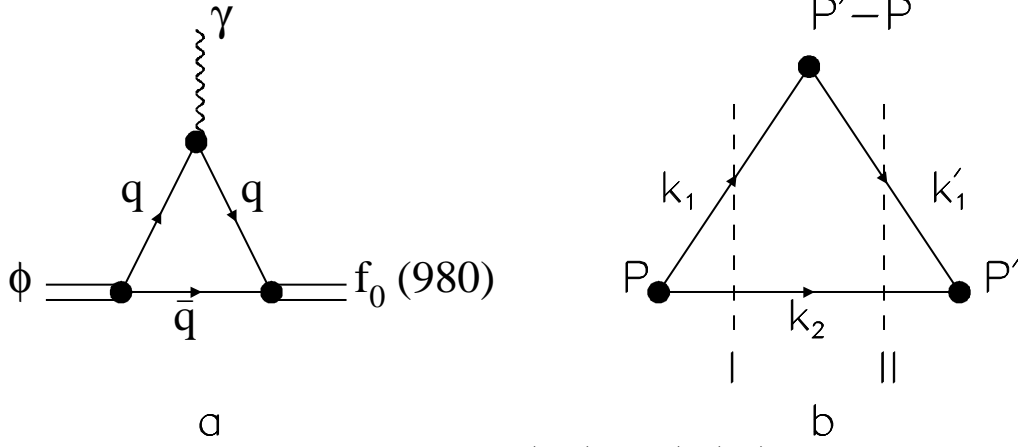


FIG. 1. a) Diagrammatic representation of the transition $\phi(1020) \rightarrow \gamma f_0(980)$. b) Three-point quark diagram: dashed lines I and II mark the two cuttings in the double spectral representation.

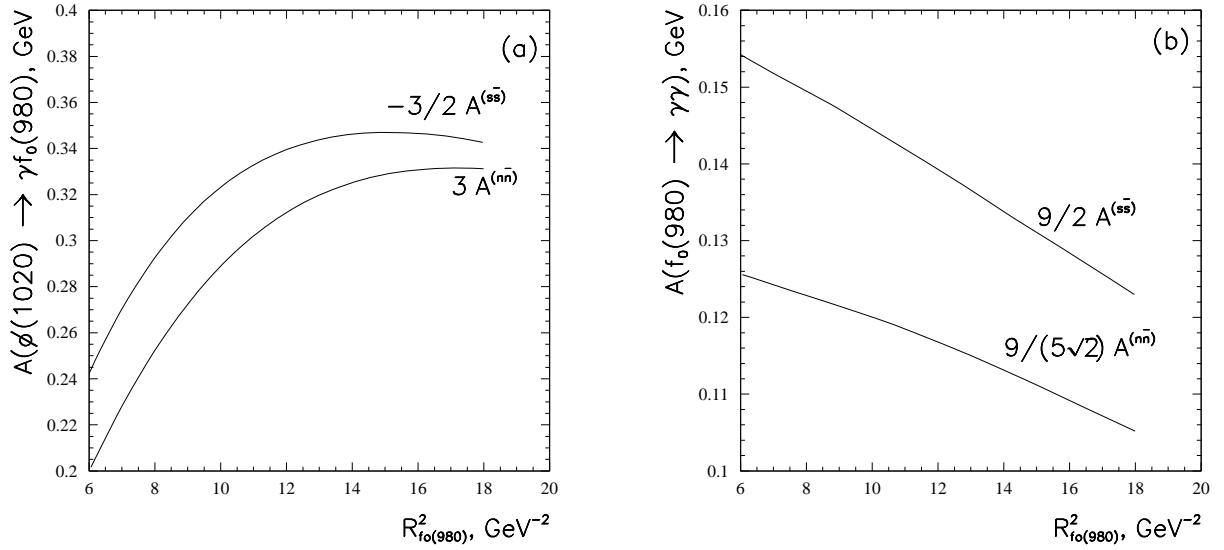


FIG. 2. Amplitudes for strange and non-strange components, $s\bar{s}$ and $n\bar{n}$, as functions of the $f_0(980)$ -meson radius squared: a) $A_{\phi \rightarrow \gamma f_0}^{(n\bar{n})}(0)/Z_{\phi \rightarrow \gamma f_0}^{(n\bar{n})}$ and $A_{\phi \rightarrow \gamma f_0}^{(s\bar{s})}(0)/Z_{\phi \rightarrow \gamma f_0}^{(s\bar{s})}$, b) $A_{\phi \rightarrow \gamma\gamma}^{(n\bar{n})}(0)/Z_{\phi \rightarrow \gamma\gamma}^{(n\bar{n})}$ and $A_{\phi \rightarrow \gamma\gamma}^{(s\bar{s})}(0)/Z_{\phi \rightarrow \gamma\gamma}^{(s\bar{s})}$.

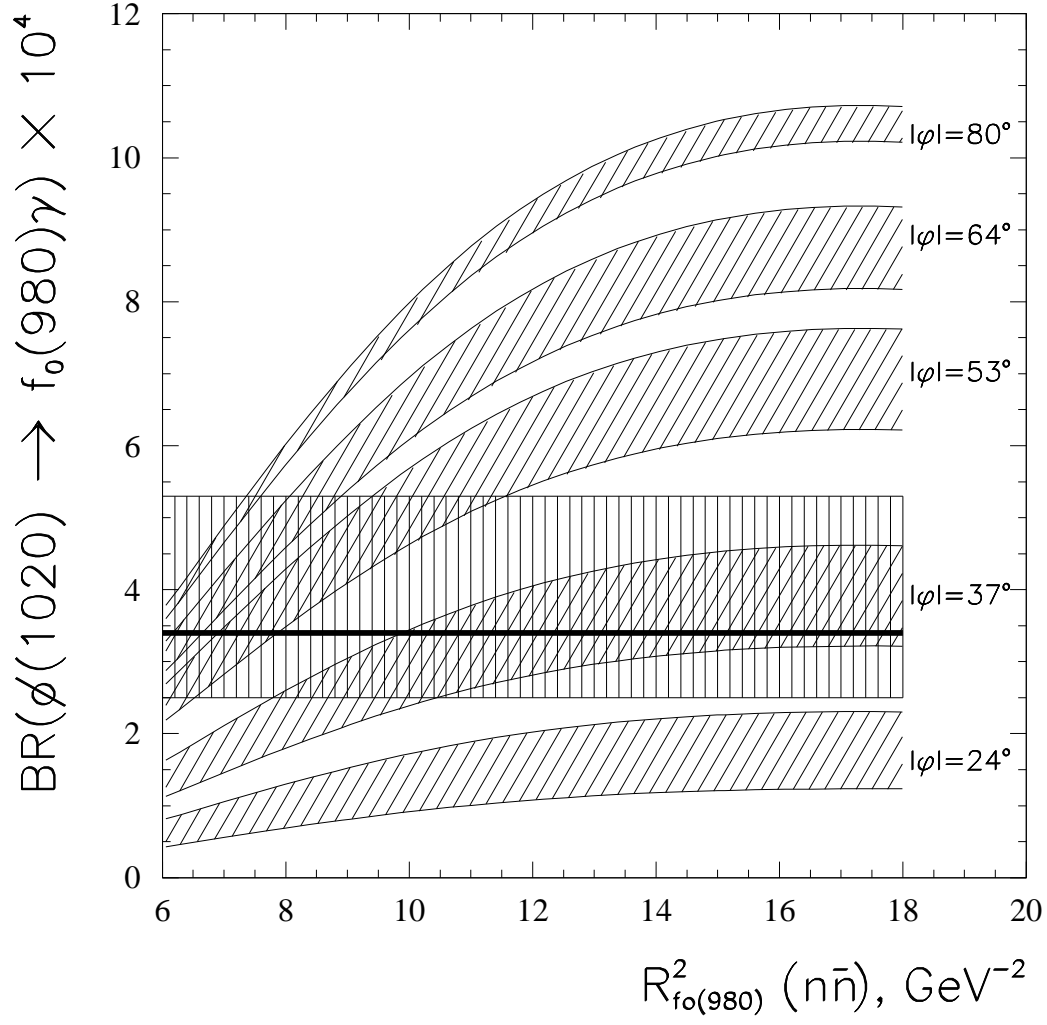


FIG. 3. Branching ratio $BR(\phi(1020) \rightarrow \gamma f_0(980))$ as a function of the radius squared of the $n\bar{n}$ component in $f_0(980)$. The band with vertical shading stands for the experimental magnitude; five other bands, with skew shading, correspond to $|\varphi| = 24^\circ, 37^\circ, 53^\circ, 64^\circ, 80^\circ$ at $-8^\circ \leq \varphi_V \leq 8^\circ$.

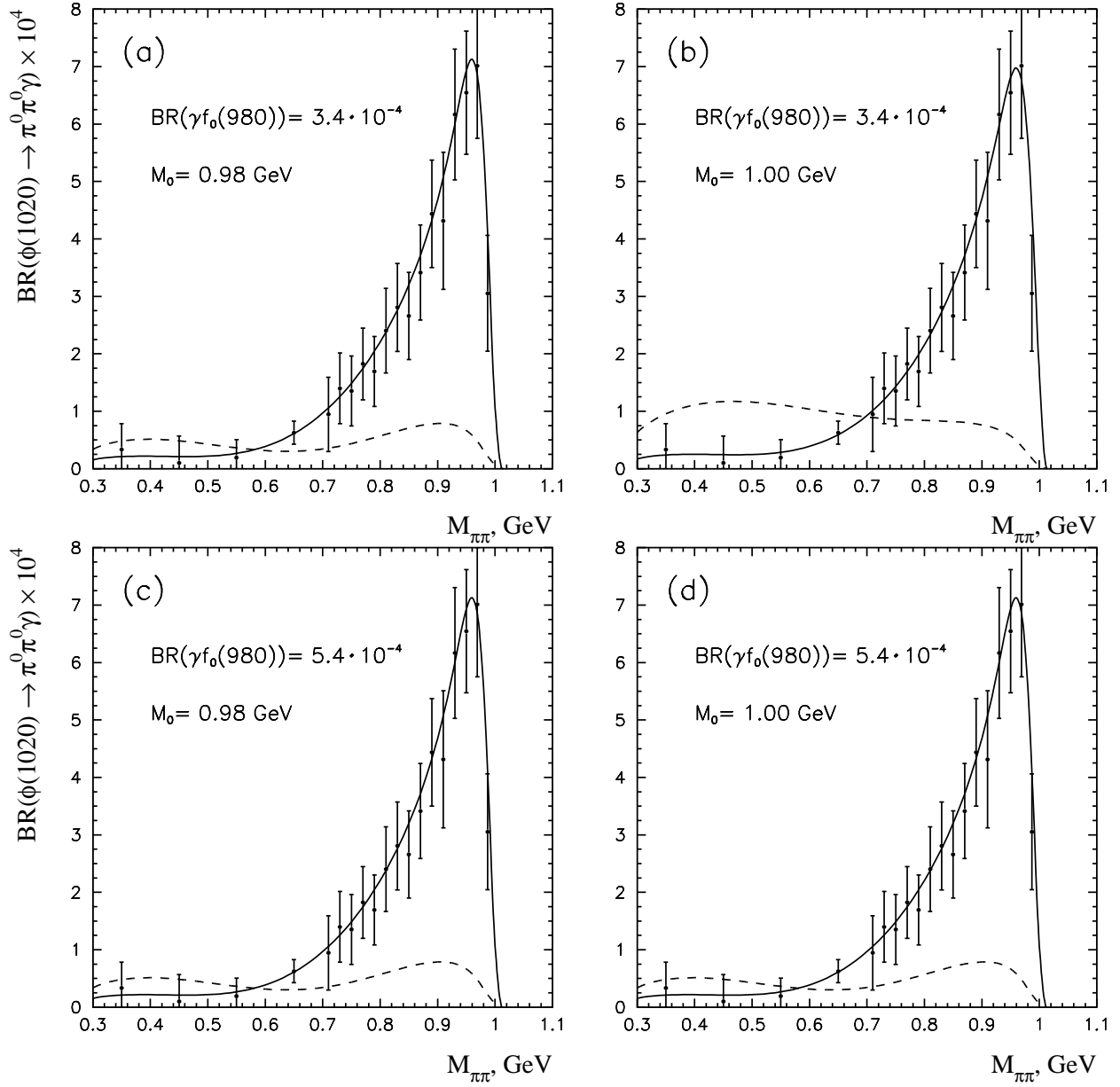


FIG. 4. Branching ratios $BR(\phi(1020) \rightarrow \pi^0 \pi^0 \gamma)$ calculated with the Flatté formula (26) and parametrization given by (28) (solid curves) at different $BR(\phi(1020) \rightarrow \gamma f_0(980))$ and $M_{f_0(980)}$. Dashed line shows the background contribution which corresponds to the elimination of the resonance term in (39). The data are taken from [7].

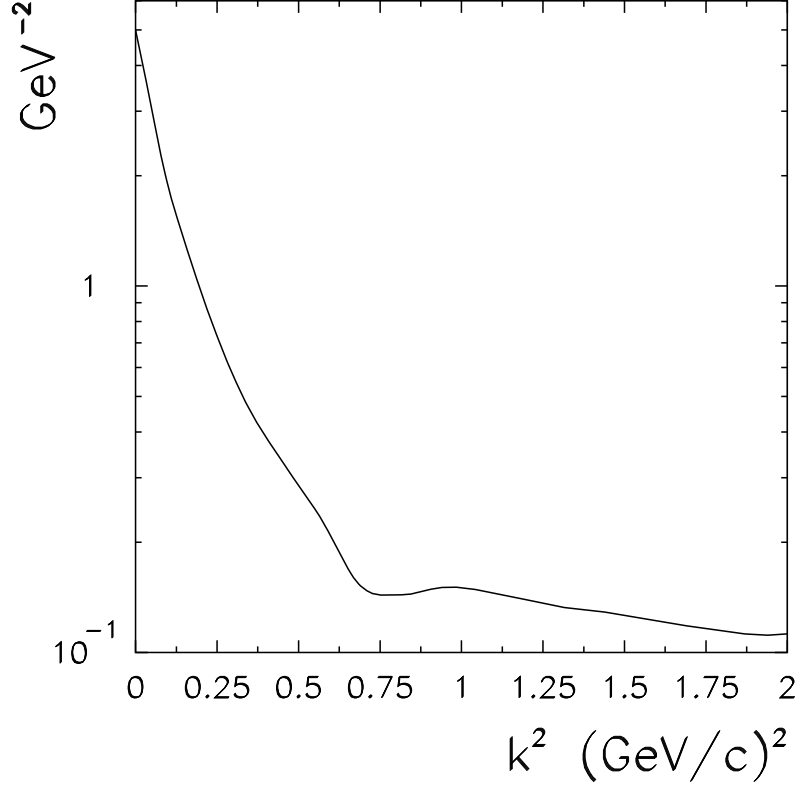


FIG. 5. Photon wave function for non-strange quarks, $\psi_{\gamma \rightarrow n\bar{n}}(k^2) = g_\gamma(k^2)/(k^2 + m^2)$, where $k^2 = s/4 - m^2$; the wave function for the $s\bar{s}$ component is equal to $\psi_{\gamma \rightarrow s\bar{s}}(k^2) = g_\gamma(k^2)/(k^2 + m_s^2)$; the constituent quark masses are $m=350$ MeV and $m_s=500$ MeV.

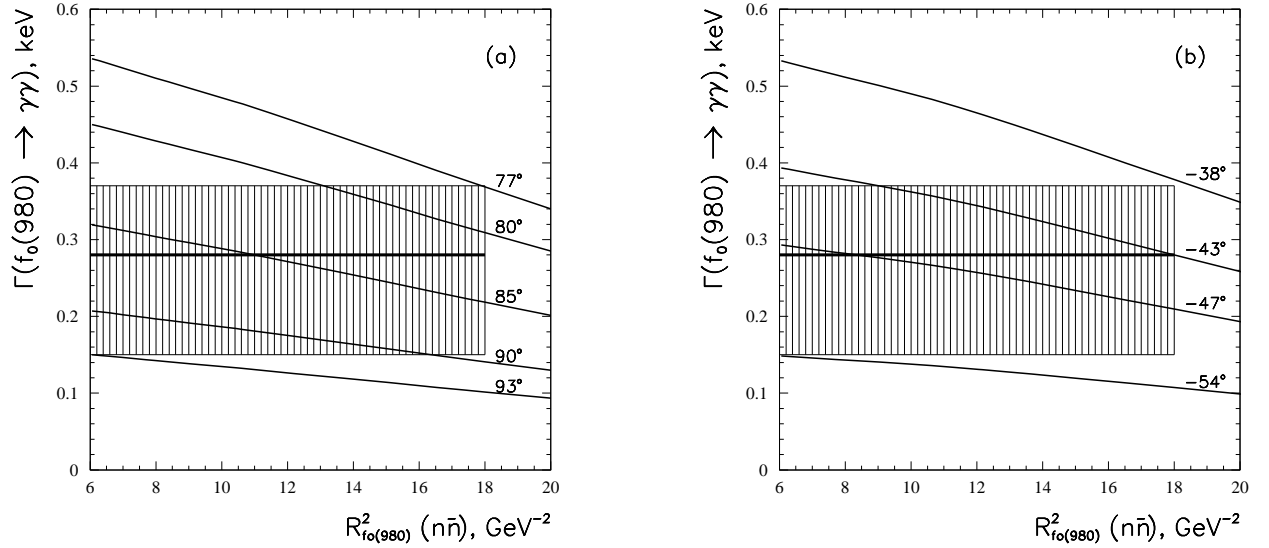


FIG. 6. Partial width $\Gamma_{f_0(980) \rightarrow \gamma\gamma}$; experimental data are from [22] (shaded area). a) Curves are calculated for positive mixing angles $\varphi = 77^\circ, 80^\circ, 85^\circ, 90^\circ, 93^\circ$ and b) negative angles $\varphi = -38^\circ, -43^\circ, -47^\circ, -54^\circ$.

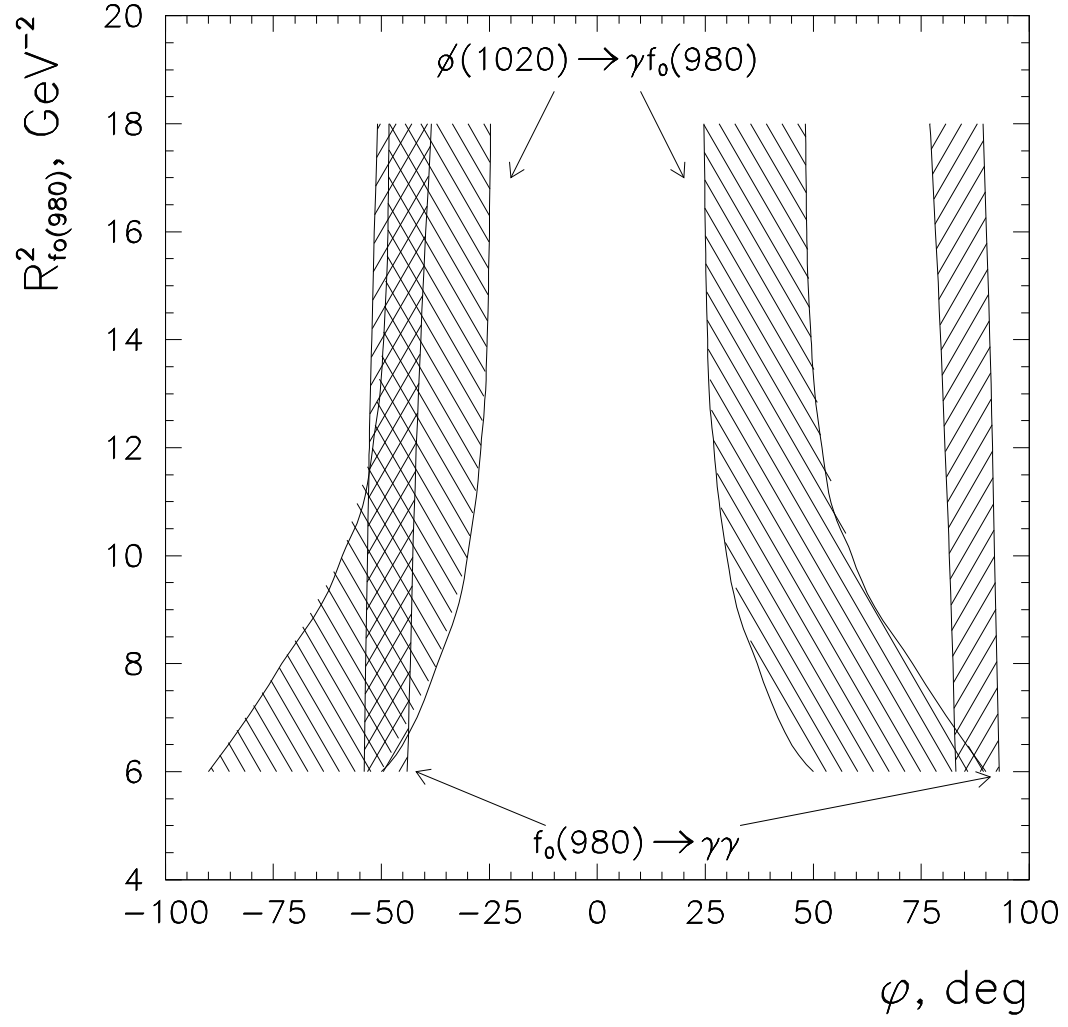


FIG. 7. The $(\varphi, R^2_{f_0(980)})$ -plot: the shaded areas are the allowed ones for the reactions $\phi(1020) \rightarrow \gamma f_0(980)$ and $f_0(980) \rightarrow \gamma\gamma$.